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Article

Comparative Study on Required Bit Depth of Gamma Quantization for Digital Cinema using Contrast and Color Difference Sensitivities

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Abstract

A specification for digital cinema systems which deal with movies digitally from production to delivery and projection on the screens is recommended by DCI (Digital Cinema Initiative), and systems based on this specification have been developed and installed in theaters. The parameters of the systems that play an important role in determining image quality include image resolution, quantization bit depth, color space, gamma characteristics, and data compression methods. This paper comparatively discusses a relation between required bit depth and gamma quantization using both of a human visual system for grayscale images and a color difference model for color images. The required bit depth obtained from a contrast sensitivity function against grayscale images monotonically decreases as the gamma value increases, while it has a minimum value when the gamma is 2.9 to 3.0 from CIE 1976 $L^*a^*b^*$ color difference model. It is also shown that the bit depth derived from the contrast sensitivity function is one bit greater than that derived from the $L^*a^*b^*$ color difference model at the gamma value of 2.9.

Key Words: *digital cinema*, *human visual system*, *contrast sensitivity function*, *quantization bit depth*, *color difference*, *CIE 1976 $L^*a^*b^*$*

1. INTRODUCTION

DCI (Digital Cinema Initiative) has decided to employ a bit depth of 12 bits and a gamma value of 2.6 for images in DCDM (Digital Cinema Distribution Master) [1]. These specified values are derived from the result obtained by using Barten's contrast sensitivity function for grayscale images [3] as discussed in [2]. However, it is not necessarily clear about the optimum gamma value because the required bit depth is experimentally determined after the gamma value, γ, is primarily determined as 2.6. Kennel [3] gives the following two points as the reason why the gamma value has been decided as 2.6.

(1) DLP (Digital Light Processing) cinema projectors had been using $\gamma = 2.6$ for several years with success. This gamma value was selected through practical testing in mastering process.

(2) The minimum modulation function with gamma quantization matches the slope of the reciprocal of the Barten's contrast sensitivity function with $1/\gamma = 1/2.6$ over the luminance range found in motion pictures.

Although the reason (1) seems to be supported by a subjective evaluation experiment to decide $\gamma = 2.6$ of the DLP projector, it doesn't seem necessary in the reason (2) to adjust the slopes derived from both the Barten's contrast sensitivity function and gamma quantization. It is based on the fact that, in order to avoid pseud-contours resulted from the luminance quantization, the luminance differences created by one quantization step change should be within one JND (Just

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Noticeable Difference) over the entire dynamic range rather than the slope of gamma quantization matches to that derived from the HVS (Human Visual System).

On the other hand, we proposed how to decide the gamma value to make sure that the color difference artifact will not occur in the CIE 1976 $L^*a^*b^*$ uniform color space and a calculation method of the required quantization bit depth [5]. According to this method, the optimum gamma value is derived from a numerical evaluation, and the required quantization bit depth can be expressed by an equation if the signal dynamic range and a perceptual threshold of the color difference are supplied.

This paper comparatively discusses the optimum gamma values and the required gamma quantization bit depths obtained from both the Barten and the CIE 1976 $L^*a^*b^*$ color difference models, and moreover shows the relationship between them. First of all, the Barten's contrast sensitivity function for grayscale images and the color difference resulted from the gamma quantization in the CIE 1976 $L^*a^*b^*$ color space for color images are reviewed in Section 2 for preparing the following discussion. Then, in Section 3, the visual modulation function for the gamma quantization is derived to compare the same modulation obtained from the Barten model, and the required bit depth for grayscale images is computed by a numerical method in such a way that the modulation from the gamma quantization is always smaller than that from the Barten model over the predefined luminance dynamic range. In Section 4, the required quantization bit depth for color images is computed numerically when the $L^*a^*b^*$ color difference model is applied, and this result is compared with that derived in Section 3. Conclusions are given in Section 5.

2. BASIC EQUATIONS

2.1 Visual modulation from Barten model

The Barten's contrast sensitivity function [3] is given by Eq. (1). The following explanation of Eq. (1) is based on [2].

$$CSF(u) = \frac{M_{opt}(u)/k}{\sqrt{\frac{2}{T}\left(\frac{1}{X_0^2}+\frac{1}{X_{\max}^2}\frac{u^2}{N_{\max}^2}\right)\left(\frac{1}{\eta p E}+\frac{\Phi_0}{1-e^{-(u/u_0)^2}}\right)}}, \tag{1}$$

$M_{opt}(u)$ is the optical MTF (Modulation Transfer Function):

$$M_{opt}(u) = e^{-2\pi^2\sigma^2u^2}, \tag{2}$$

σ is the standard deviation of the line-spread function

$$\sigma = \sqrt{\sigma_0^2 + \left(C_{ab}d\right)^2}, \tag{3}$$

σ_0 is a constant, C_{ab} is the spherical aberration of the eye, and d is the pupil diameter described as

$$d = 5 - 3\tanh\left(0.4\log\left(LX_0^2/40^2\right)\right), \tag{4}$$

E is the retinal illuminance in Trolands. The equation for E is

$$E = \frac{\pi d^2}{4}L\left\{1-\left(\frac{d}{9.7}\right)^2+\left(\frac{d}{12.4}\right)^4\right\}. \tag{5}$$

In Eq. (1), other parameters are defined as follows.

- k : signal-to-noise ratio of the eye,
- T : integration time of the eye,
- X_0 : angular size of the object image,
- $X_{\max}$: maximum angular size of the integration area,
- $N_{\max}$: maximum number of cycles over which the eye can integrate the information,
- η : quantum efficiency of the eye,
- p : photon conversion factor,
- Φ_0 : spectral density of neural noise,
- u_0 : maximum frequency of lateral inhibition.

The parameters that Barten recommended for the calculation are listed below. It should be noted that Barten did not recommend a particular value of X_0 to use.

k = 3.0, T = 0.1 (sec), η = 0.03, σ_0 = 0.0083 (arc deg), $X_{\max}$ = 12 (deg), $\Phi_0 = 3 \times 10^{-8}$ (sec deg^2), C_{ab} = 0.0013 (arc deg/mm), $N_{\max}$ = 15 (cycles), u_0 = 7 (cycles/deg).

In [2], it is agreed to use a value of 1.285×10^6 photons/sec/deg^2/Td as the value of the parameter p. Therefore, the factors that can be varied in a calculation are the frequencies, u, the luminance, L, and the field of view, X_0. The product of u and X_0 gives the number of cycles in a pattern, and it is defined in the experiment of [2] as 13 cycles. Therefore, X_0 can be made a variable dependent on u:

$$X_0 = \frac{13}{u}. \tag{6}$$

Thus, Eq. (1) can be reduced to two independent variables, the luminance, L, and the spatial frequency, u.

By assuming that the reciprocal of Eq. (1) is a visual modulation threshold to avoid pseud-contour in images, it can be defined that a luminance change corresponding to one quantization step should be smaller than this threshold, $m = 1/CSF(u)$. This condition often refers to that a luminance step induced by quantization is lower than one JND. Because $m = 1/CSF(u)$ is a function of u, the modulation threshold can be obtained by the numerical calculation method: firstly we define a luminance value, L, and secondly we compute u that maximizes $CSF(u)$ by using Eq. (1). As a result, the modulation threshold, m, is obtained from the reciprocal of the maximum contrast sensitivity function, $CSF_{\max}(u)$.

In the following evaluation, the aim luminance range is set from 0.0041 to 41 cd/m^2 (i.e., 10,000:1 dynamic range) by considering the range used for the projection of motion pictures in a theater. Figure 1 shows results of (a) $u_{\max}$ that maximizes the contrast sensitivity function, $CSF(u)$, and (b) $m = 1/CSF_{\max}(u)$ at $u = u_{\max}$.

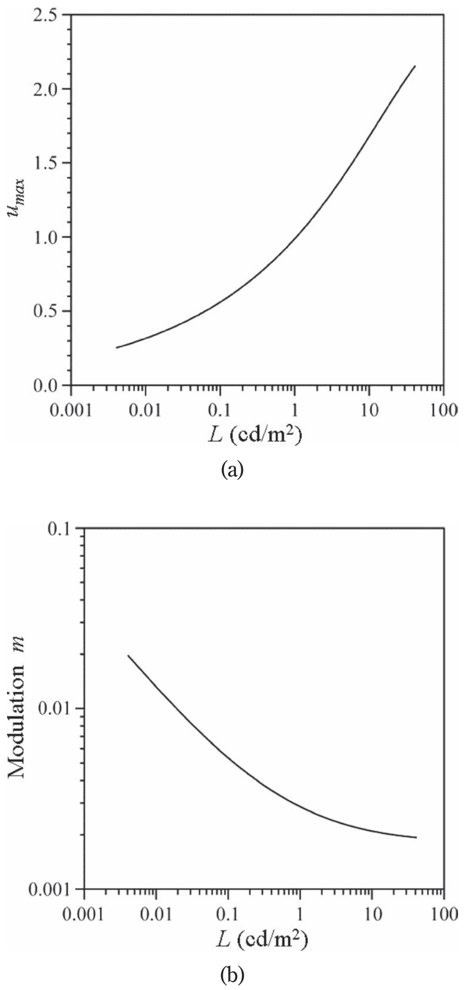


Fig. 1 Results from Barten model. (a) L and $u_{\max}$, (b) L and m.

2.2 CIE 1976 $L^*a^*b^*$ color difference model

In the DCI specification [1], the CIE XYZ tri-stimulus values are calculated with a normalizing constant that sets the Y tri-stimulus value equal to the absolute luminance in cd/m^2. The encoding transfer function is defined by

$$\left.\begin{aligned} k_x &= INT\left[4095\left(\frac{X}{52.37}\right)^{1/2.6}\right] \\ k_y &= INT\left[4095\left(\frac{Y}{52.37}\right)^{1/2.6}\right] \\ k_z &= INT\left[4095\left(\frac{Z}{52.37}\right)^{1/2.6}\right], \end{aligned}\right\} \tag{7}$$

where k_x, k_y, k_z ($0 \le k_x, k_y, k_z, \le 4095$) are quantization codes of XYZ tri-stimulus values, and the function $INT[s]$ returns the nearest integer value of s. Therefore, the bit depth of 12 bits and the gamma value of 2.6 are defined for the gamma quantization of color images. Although the XYZ tri-stimulus values are normalized by 52.37 (cd/m^2) which is greater than the value of maximum luminance of 41 (cd/m^2) assumed in Section 2.1, this includes extra headroom reserved to accommodate a range of various white points such as D_{55}, D_{61} and D_{65} light sources.

A smallest required bit depth can be obtained in such a way that the maximum color difference of adjacent quantized color values always becomes smaller than the predetermined threshold of allowable color difference in the CIE 1976 $L^*a^*b^*$ uniform color difference space. Let the quantized values of X, Y and Z normalized their maximum values be $x(k_x)$, $y(k_y)$ and $z(k_z)$, the corresponding values of L^*, a^*, and b^* at the coordinate (k_x, k_y, k_z) in the CIE 1976 $L^*a^*b^*$ space are defined by

$$\left.\begin{aligned} L^*\left(k_x, k_y, k_z\right) &= 116 f\left(y(k_y)\right) - 16 \\ a^*\left(k_x, k_y, k_z\right) &= 500\left\{f\left(x(k_x)\right) - f\left(y(k_y)\right)\right\} \\ b^*\left(k_x, k_y, k_z\right) &= 200\left\{f\left(y(k_y)\right) - f\left(z(k_z)\right)\right\}, \end{aligned}\right\} \tag{8}$$

where the function $f(w)$ is defined as follows.

$$f(w) = \begin{cases} w^{1/3}, & if\ (24/116)^3 < w \le 1 \\ (1/3)(116/24)^2 w + (16/116), & if\ w \le (24/116)^3. \end{cases} \tag{9}$$

If we assume that the quantization codes k_x, k_y and k_z

in the $L^*a^*b^*$ color space change by δ_x, δ_y, and δ_z (these values are either –1, 0 or +1), respectively, the $L^*(k_x, k_y, k_z)$, $a^*(k_x, k_y, k_z)$, and $b^*(k_x, k_y, k_z)$ values can be written as $L^*(k_x+\delta_x, k_y+\delta_y, k_z+\delta_z)$, $a^*(k_x+\delta_x, k_y+\delta_y, k_z+\delta_z)$, and $b^*(k_x+\delta_x, k_y+\delta_y, k_z+\delta_z)$. Then, the quantized step sizes of L^*, a^*, and b^* are

$$\left.\begin{aligned}\Delta L^*(k_x, k_y, k_z) &= L^*(k_x+\delta_x, k_y+\delta_y, k_z+\delta_z) - L^*(k_x, k_y, k_z)\\ \Delta a^*(k_x, k_y, k_z) &= a^*(k_x+\delta_x, k_y+\delta_y, k_z+\delta_z) - a^*(k_x, k_y, k_z)\\ \Delta b^*(k_x, k_y, k_z) &= b^*(k_x+\delta_x, k_y+\delta_y, k_z+\delta_z) - b^*(k_x, k_y, k_z).\end{aligned}\right\} \tag{10}$$

Consequently, the color difference, ΔEab, is obtained as follows by computing the Euclidean distance between two points that are separated by ΔL^*, Δa^*, and Δb^* in L^*, a^*, and b^* directions,

$$\begin{aligned}\Delta Eab &= \left[\left\{\Delta L^*(k_x, k_y, k_z)\right\}^2 + \left\{\Delta a^*(k_x, k_y, k_z)\right\}^2 + \left\{\Delta b^*(k_x, k_y, k_z)\right\}^2\right]^{1/2}\\ &= \Big[A\left\{f\left(y(k_y+\delta_y)\right) - f\left(y(k_y)\right)\right\}^2\\ &\quad + B\left\{f\left(x(k_x+\delta_x)\right) - f\left(y(k_y+\delta_y)\right) - \left(f\left(x(k_x)\right) - f\left(y(k_y)\right)\right)\right\}^2\\ &\quad + C\left\{f\left(y(k_y+\delta_y)\right) - f\left(z(k_z+\delta_z)\right) - \left(f\left(y(k_y)\right) - f\left(z(k_z)\right)\right)\right\}^2\Big]^{1/2},\end{aligned} \tag{11}$$

where $A = 116^2$, $B = 500^2$, and $C = 200^2$. The values of $x(k_x)$, $y(k_y)$, and $z(k_z)$ are determined by the quantization function (i.e., gamma quantization characteristics), and the specific expression for the gamma quantization is discussed in Section 4. We expect any pseud-contours will not occur in images if the color difference, ΔEab, which is produced by the gamma quantization as Eq. (11), is smaller than one JND. In this paper, we employed the threshold as $\Delta Eab = 1$.

3. REQUIRED BIT DEPTH FROM BARTEN MODEL

3.1 Gamma quantization of entire luminance range

A visual modulation for a sinusoidal luminance pattern, m, is given as follows when L_{high} and L_{low} are highest and lowest luminance of the pattern, respectively [2].

$$m = \frac{L_{high} - L_{low}}{L_{high} + L_{low}}. \tag{12}$$

Moreover, let ΔL be a difference between L_{high} and L_{low}, and L_{ave} be an average of L_{high} and L_{low}:

$$\Delta L = L_{high} - L_{low}, \tag{13}$$

$$L_{high} + L_{low} = 2L_{ave}. \tag{14}$$

Substituting Eq. (13) and Eq. (14) into Eq. (12), we obtain

$$m = \frac{\Delta L}{2L_{ave}}. \tag{15}$$

The visual modulation, m, is determined by the representation of ΔL (i.e., gamma quantization). Firstly, we consider the required bit depth of the gamma quantization for which the minimum quantized luminance is 0 and the maximum quantized luminance is P considering the input luminance (L) range is defined by $0 \le L \le P$. Thus, the quantized luminance is given by

$$L = P\left(\frac{k}{2^n - 1}\right)^\gamma, \tag{16}$$

where n is a quantization bit depth, k is a code value of the gamma quantization ($0 \le k \le 2^n-1$), and γ is a gamma value. Therefore, the luminance step, ΔL, corresponding to one code value difference is

$$\Delta L = \frac{P}{\left(2^n - 1\right)^\gamma}\left\{(k+1)^\gamma - k^\gamma\right\}, \tag{17}$$

and the luminance average, L_{ave}, of the two luminance values that have one code value difference is

$$L_{ave} = \frac{P}{2\left(2^n - 1\right)^\gamma}\left\{(k+1)^\gamma + k^\gamma\right\}. \tag{18}$$

Substituting Eq. (17) and Eq. (18) into Eq. (15), we obtain

$$m = \frac{(k+1)^\gamma - k^\gamma}{(k+1)^\gamma + k^\gamma}. \tag{19}$$

The visual modulation threshold, m, is computed using Eq. (19) for gamma values of 1.5, 2.0 2.6 and 3.0, and the results are shown in Fig. 2 together with that derived from the Barten model. In a relatively high luminance region, k is sufficiently greater than 1 for generally $2^n \gg 1$, so that the following approximation can be derived for $k \gg 1$

$$(k+1)^\gamma = k^\gamma + \gamma k^{\gamma-1}, \tag{20}$$

and this leads to a modification of the visual modulation threshold, m, as,

$$m = \frac{\gamma}{2k + \gamma}. \tag{21}$$

Because we are considering m at the relatively high luminance region ($k \gg \gamma$), Eq. (21) are approximately

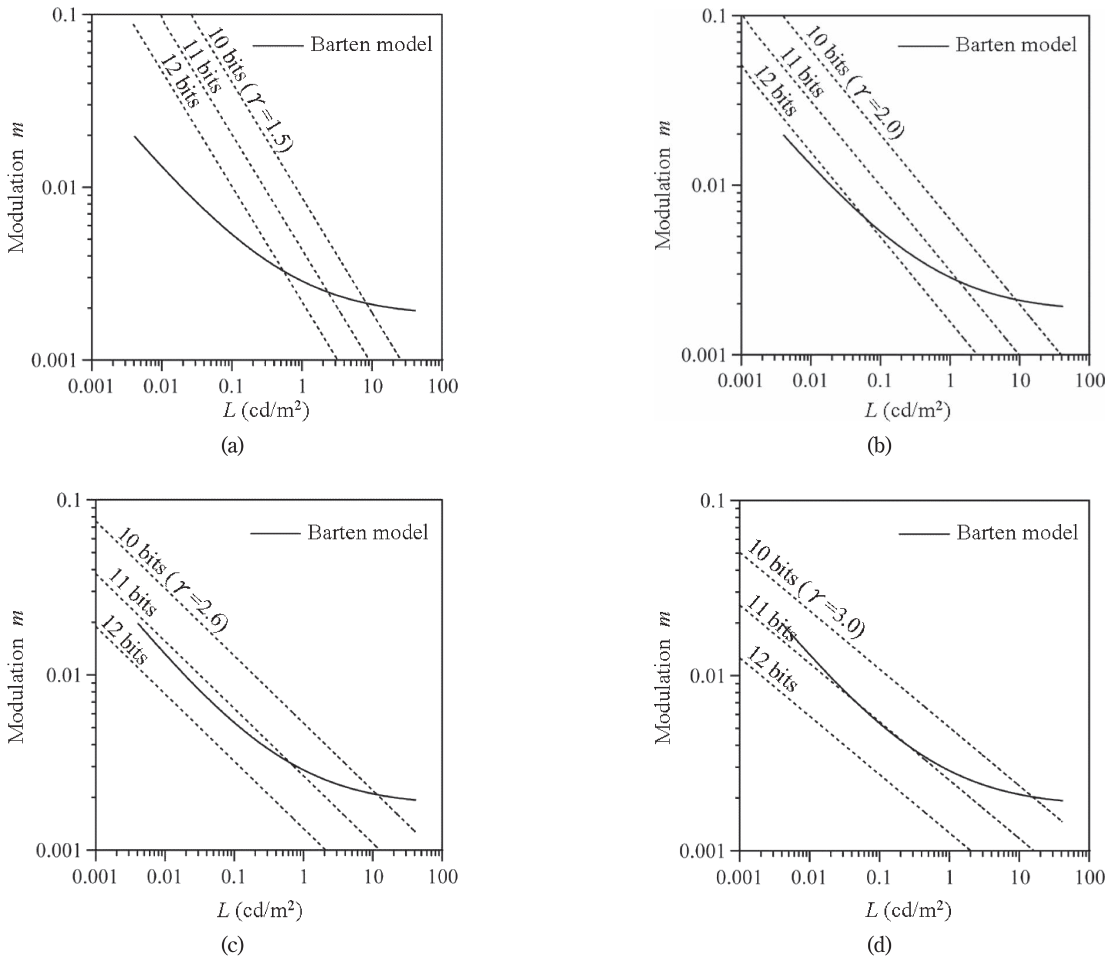


Fig. 2 Visual modulation threshold for gamma quantization with $\rho = 0$ (10, 11, 12 bits). (a) $\gamma = 1.5$, (b) $\gamma = 2.0$, (c) $\gamma = 2.6$, (d) $\gamma = 3.0$.

represented by

$$m \approx \frac{\gamma}{2k}. \tag{22}$$

On the other hand, k is derived from Eq. (16) as

$$k = \left(2^n - 1\right)\left(\frac{L}{P}\right)^{1/\gamma}. \tag{23}$$

Substituting this k into Eq. (22), we obtain

$$m = \frac{\gamma}{2\left(2^n - 1\right)}\left(\frac{L}{P}\right)^{-1/\gamma}. \tag{24}$$

This equation can be represented as follows in the logarithmic domain.

$$\log m = -\frac{1}{\gamma}\log L + \log\frac{\gamma P^{1/\gamma}}{2\left(2^n - 1\right)} \tag{25}$$

Eq. (25) shows that m and L have a linearly changing relationship in the logarithmic coordinate and the slop of the line is $-1/\gamma$ in the relatively high luminance region. When both the quantization bit depth, m, and the highest luminance value, P, are constant, the line of m against L moves downward as γ increases because the decrement by $P^{1/\gamma}$ is greater than the increment of γ in the second term of the right side in Eq. (25). The simulation results in Fig. 2 illustrate that this relationship is maintained.

Next, we consider the minimum bit depth for a given gamma value. This problem corresponds to finding a minimum bit depth with maintaining the graph of m for the gamma quantization always below that for the Barten model for a given γ within the concerned luminance range in Fig. 2. In practice, the required bit depth should be an integer number, but we will search for it as a real number. To determine the required minimum bit depth, we perform the following steps:

1. Determine γ.
2. Determine an initial bit depth of integer value (e.g., 8 bits), and set the bit precision $\Delta = 1$.
3. Compute the visual modulation threshold, m_q, for the gamma quantization using Eq. (19) for k from 0 to 2^n-1.
4. Compute the average luminance value, L_{ave}, using Eq. (18) for the k.
5. Compare m_q and the visual modulation threshold,

m_b, from the Barten model for L_{ave}. If $m_q < m_b$ for all L_{ave}, then go to Step 6. If not, then go back to Step 3 after setting $n = n + \Delta$.

6. If $\Delta = 0.001$, then this procedure is ended (i.e., the precision of bit depth we found have 3 digits below decimal point). If not, go back to Step 3 after setting $n = n - \Delta$, $\Delta = \Delta/10$, and $n = n + \Delta$.

The result obtained by the above procedure is shown in Fig. 3 (a). This result coincides to that from Fig. 2, and it is obvious that the required bit depth decreases as γ increases. Fig. 3 (b) shows a critical luminance (i.e., a luminance that determines the required bit depth), and the critical luminance shifts to higher luminance as γ increases.

3.2 Gamma quantization of limited luminance range

In this subsection, we consider a case that the gamma quantization is performed according to the dynamic range of the grayscale image. The quantized luminance, L, is given by

$$L = \rho + (P - \rho)\left(\frac{k}{2^n - 1}\right)^\gamma, \tag{26}$$

where n is a quantization bit depth, k is a code value of the gamma-quantization ($0 \le k \le 2^n - 1$), ρ is a lowest luminance value, P is a highest luminance value, and γ is a gamma value of the quantizer. Therefore, the luminance step, ΔL, corresponding to one code value difference is

$$\Delta L = \frac{P - \rho}{\left(2^n - 1\right)^\gamma}\left\{(k+1)^\gamma - k^\gamma\right\}, \tag{27}$$

and the luminance average, L_{ave}, of the two luminance values that have one code value difference is

$$L_{ave} = \frac{P - \rho}{2\left(2^n - 1\right)^\gamma}\left\{(k+1)^\gamma + k^\gamma\right\} + \rho. \tag{28}$$

Substituting Eq. (27) and Eq. (28) into Eq. (15), we obtain

$$m = \frac{(k+1)^\gamma - k^\gamma}{(k+1)^\gamma + k^\gamma + \dfrac{2\rho}{P - \rho}\left(2^n - 1\right)^\gamma}. \tag{29}$$

If the dynamic range of the luminance is represented as $Dr = P/\rho$, the visual modulation threshold can be written by

$$m = \frac{(k+1)^\gamma - k^\gamma}{(k+1)^\gamma + k^\gamma + 2\left(2^n - 1\right)^\gamma / (Dr - 1)}. \tag{30}$$

Eq. (30) coincides to Eq. (19) when the lowest luminance value, ρ, is 0 (i.e., $Dr = \infty$), so that the visual modulation threshold decreases with a slop of $-1/\gamma$ in the relatively high luminance range as seen in subsection 3.1 when the dynamic range of the luminance is sufficiently high.

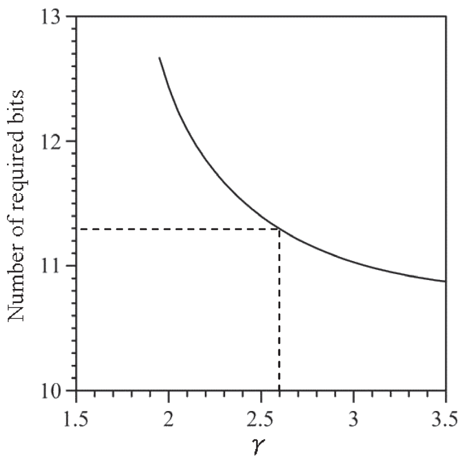
The behavior of the visual modulation threshold, m, can be investigated by regarding m as a function of the quantization code, k, that is,

$$m = \frac{h(k)}{g(k)}. \tag{31}$$

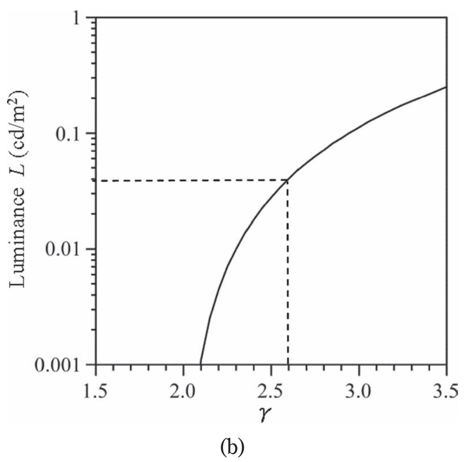
From Eq. (30), $h(k)$ and $g(k)$ are the functions of k as follows.

$$h(k) = (k+1)^\gamma - k^\gamma, \tag{32}$$

$$g(k) = (k+1)^\gamma + k^\gamma + a. \tag{33}$$



(a)



(b)

Fig. 3 Gamma value dependence with Barten model and gamma quantization with $\rho = 0$. (a) required bit depth, (b) critical luminance.

where a is a variable independent from k:

$$a=\frac{2(2^n-1)^\gamma}{Dr-1}. \tag{34}$$

The fluctuation of dm/dk is determined by

$$\begin{aligned}q(k)&=h'(k)g(k)-h(k)g'(k)\\&=\gamma\left[-2k^{\gamma-1}(k+1)^{\gamma-1}+a\left\{(k+1)^{\gamma-1}-k^{\gamma-1}\right\}\right].\end{aligned} \tag{35}$$

It is difficult to obtain a general solution analytically for $q(k)=0$ because γ is a real number. If $k=0$ (i.e., lowest luminance value),

$$q(0)=\gamma a>0 \tag{36}$$

is always satisfied for $\gamma>1$, so that the visual modulation threshold, m, is an increasing function at the lowest luminance.

According to the above discussion, it becomes obvious that the visual modulation threshold, m, is increasing function at the lowest luminance and also is decreasing function at relatively high luminance. Therefore, m has at least one relative maximum within this luminance range. A simplest example to solve $q(k)=0$ analytically is the case of $\gamma=2$. In the followings, we outline the behavior of m for such a case.

Substituting $\gamma=2$ into Eq. (35), we obtain

$$h(k)=\gamma\left(-2k^2-2k+a\right). \tag{37}$$

Thus, the k that satisfies $q(k)=0$ is a root of the quadratic equation, so that the k is simply derived by solving the equation as

$$k=\frac{-1+\sqrt{1+2a}}{2}, \tag{38}$$

where k should be a positive value. From Eq. (34), a is

$$a=\frac{2\left(2^n-1\right)^2}{Dr-1} \tag{39}$$

for $\gamma=2$. Substituting Eq. (39) into Eq. (38), the k that satisfies $q(k)=0$ is

$$k=\frac{1}{2}\left(\sqrt{\frac{4(2^n-1)^2}{Dr-1}}-1\right). \tag{40}$$

The function, $q(k)$, have a maximum value at the k of Eq. (40) because $q(k)$ changes its sign from positive to negative as the k increases around the value determined by Eq. (40). The luminance, L_{max}, that gives maximum visual modulation threshold is obtained by substituting Eq. (40) into Eq. (26) and setting $\gamma=2$.

$$L_{\max}=\rho+\frac{(P-\rho)}{4(2^n-1)^2}\left(\sqrt{\frac{4(2^n-1)^2}{Dr-1}}-1\right)^2. \tag{41}$$

Now, we can compute L_{max} by using ρ = 0.0041 (cd/m^2) and P = 41 (cd/m^2) which are the same as those employed in the evaluation of the Barten model. For 10, 11 and 12 bits quantization, L_{max} derived from Eq. (41) are 0.0078, 0.0080 and 0.0081 (cd/m^2), respectively. These values are obviously near the lowest luminance, so that the visual modulation threshold, m, has a maximum value at nearly lowest luminance and decreases linearly with a slop of $-1/\gamma$ at relatively high luminance.

In the followings, Eq. (30) is evaluated by a computer simulation. The visual modulation threshold, m, is plotted for γ = 1.5, 2.0, 2.6 and 3.0 in Fig. 4 together with that obtained from the Barten model. It can be seen from the figure that m of the gamma quantization have a maximum value at relatively low luminance and decreases with a slop of $-1/\gamma$ at relatively high luminance as discussed above, so that the constraint for the required bit depth of the gamma quantization is alleviated as the gamma value increases. Figure 5 (a) shows the minimum bit depth required against gamma value. The procedure to compute this bit depth is the same as that described in subsection 3.1. This result shows that the required bit depth decreases as the gamma value increases, which coincide with the result obtained in Fig. 4 (a). Figure 5 (b) shows that the critical luminance shifts to higher luminance as monotonically as shown in Fig. 4 (b) when the gamma value increases.

4. REQUIRED BIT DEPTH FROM COLOR DIFFERENCE MODEL

Although the required bit depth can be derived from the contrast sensitivity for grayscale images as described in the previous section, it also can be obtained by color difference model such as CIE 1976 $L^*a^*b^*$ [6]. The evaluation of bit depth by using the color difference model might be a straightforward method, for the DCI specification specifies the images by CIE XYZ tri-stimulus values for the purpose of DCDM.

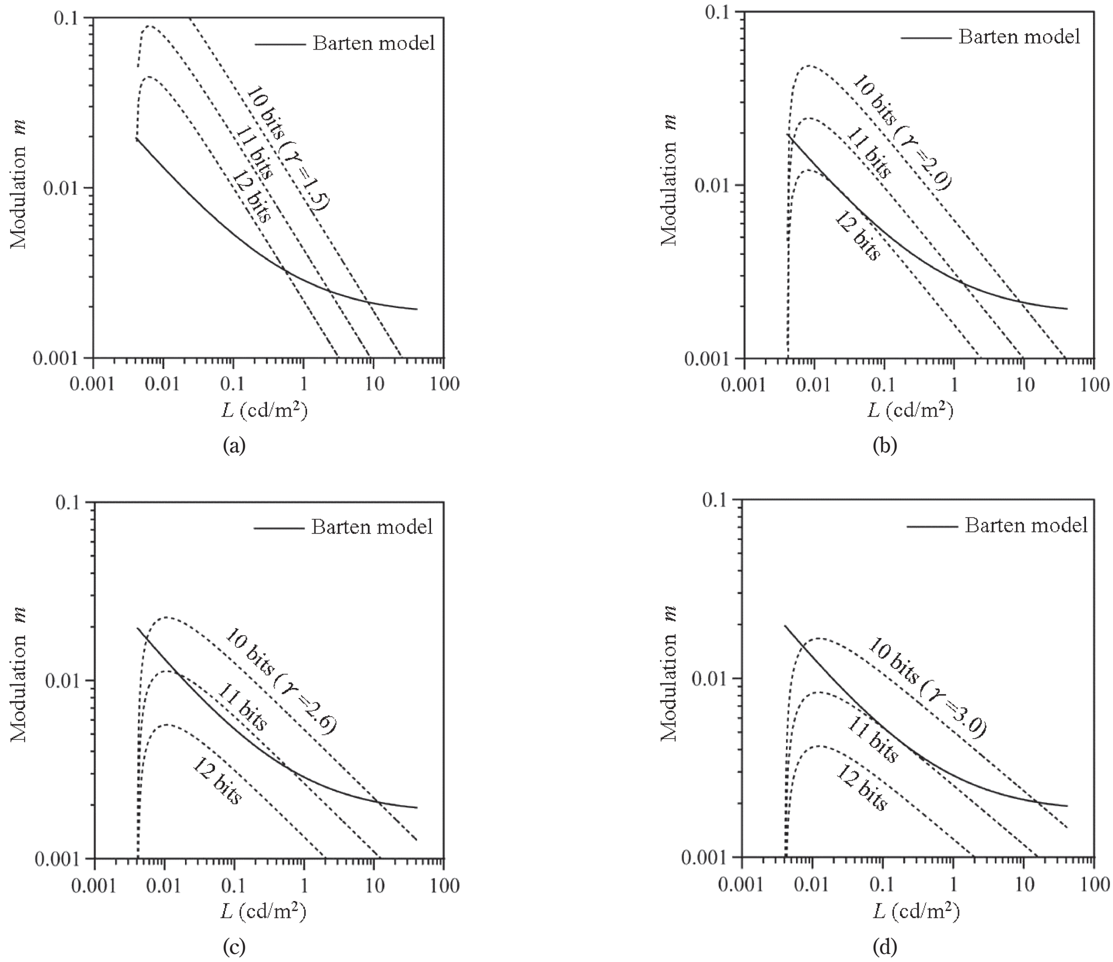


Fig. 4 Visual modulation threshold for gamma quantization with Dr = 10,000 (10, 11, 12 bits). (a) γ = 1.5, (b) γ = 2.0, (c) γ = 2.6, (d) γ = 3.0.

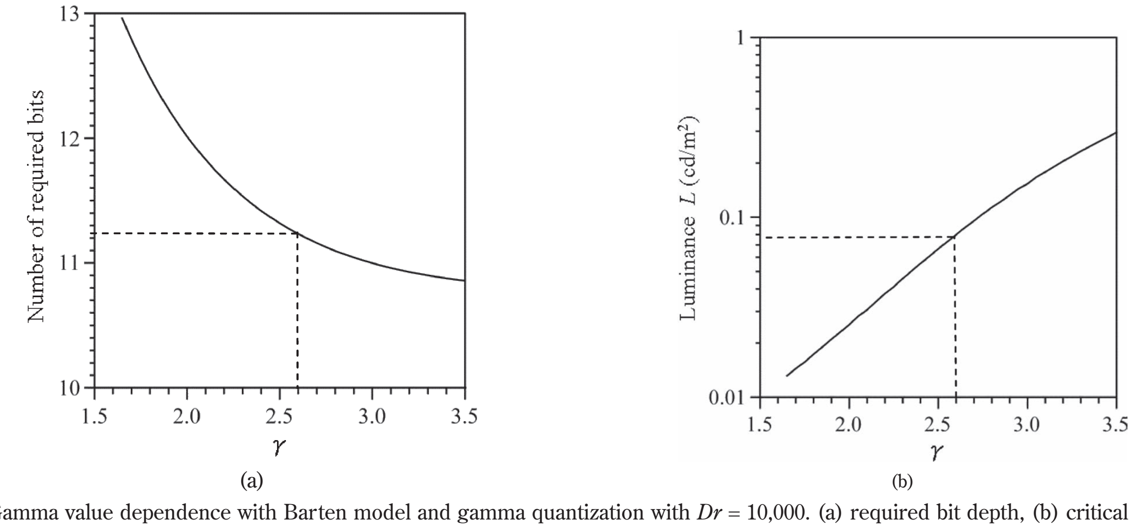


Fig. 5 Gamma value dependence with Barten model and gamma quantization with Dr = 10,000. (a) required bit depth, (b) critical luminance.

If one of the tri-stimulus values, X, is normalized by its maximum value, the gamma-corrected value of the signal, x_γ, is given by

$$x_\gamma = (x - \rho/P)^{1/\gamma}, \quad \gamma > 1, \tag{42}$$

where ρ and P are minimum and maximum values of X, respectively. This leads to the following equation by letting the signal dynamic range be $Dr = P/\rho$.

$$x = x_\gamma^\gamma + Dr^{-1}. \tag{43}$$

As the gamma-corrected quantization uniformly digitizes x ($0 \le x \le (1-Dr^{-1})^{1/\gamma}$) with n bits, the quantization step size is given by

$$\Delta = \frac{(1 - Dr^{-1})^{1/\gamma}}{2^n - 1}. \tag{44}$$

Substituting the tri-stimulus X signal into the luminance, L, in Eq. (26), we obtain

$$X = \rho + (P - \rho)\left(\frac{k_x}{2^n - 1}\right)^{\gamma}, \tag{45}$$

where k_x is a code value of the quantizer ($0 \le k_x \le 2^n - 1$). When the both sides of Eq. (45) is divided by P and the normalized quantized value of X is replaced by $x(k_x)$, Eq. (45) is rewritten by

$$x(k_x) = Dr^{-1} + (1 - Dr^{-1})\left(\frac{k_x}{2^n - 1}\right)^{\gamma}. \tag{46}$$

By using the relation of Eq. (44), this equation becomes

$$x(k_x) = Dr^{-1} + \left(\Delta k_x\right)^{\gamma}. \tag{47}$$

Because $y(k_y)$ and $z(k_z)$ are similarly obtained for the tri-stimulus Y and Z values, the color difference, ΔEab, can be calculated numerically by substituting $x(k_x)$, $y(k_y)$, and $z(k_z)$ into Eq. (11). This numerical evaluation is applied to all the quantized points in the three-dimensional color space, so that the number of operations in this calculation becomes enormous and impractical when the number of quantization bits is relatively large.

In order to solve the problem of this amount of operations, we proposed the method to compute the required number of bits for color signals analytically assuming that the change of color difference induced by the quantization in the CIE 1976 $L^*a^*b^*$ color space expressed in Eq. (11) are small enough in the continuous values of L^*, a^* and b^* [5]. As a result, the required bit depth, n, can be calculated by the following equation.

$$n = \log_2\left\{\gamma\left(1 - \rho_d\right)^{1/\gamma}\left(A + 4B + 4C\right)^{1/2}\mu\Big/\left(3\Delta Eab\right) + 1\right\} \tag{48}$$

where $\rho_d = 10^{-D}$ (D is a dynamic range of the signal when converting it into the density value: $D = \log_{10}Dr$), $\mu = w_c^{-2/3}(w_c - \rho_d)^{1-1/\gamma}$, $w_c = \{(2/3)/(1/\gamma - 1/3)\}\rho_d$. Therefore,

$$\rho_d = \frac{1}{Dr} = \frac{\rho}{P}. \tag{49}$$

Using this relationship, Eq. (48) can be rewritten by

$$n = \log_2\left\{\gamma\left(1 - 1/Dr\right)^{1/\gamma}\left(A + 4B + 4C\right)^{1/2}\mu\Big/\left(3\Delta Eab\right) + 1\right\}. \tag{50}$$

Thus, the required bit depth for the gamma quantization using the CIE 1976 $L^*a^*b^*$ color difference model can be determined from both the dynamic range of tri-stimulus XYZ signals and the gamma value, γ, if the perceptual threshold, ΔEab, is given. Although the dynamic range of $\log_{10}Dr = 3.2$, which is a typical value of the movie films [7], is employed in [5], the following evaluations for the optimum gamma value and the required bit depth extend the dynamic rang to 10,000:1 as described in Section 2.

Figure 6 shows the result of calculating the required quantization bit depth against γ based on Eq. (50) when the dynamic range, $\log_{10}Dr$, of the color signals is set to 3.0, 3.2, 3.4, 3.6, 3.8 and 4.0. Here, $\Delta Eab = 1$ is assumed to be a permissible perceptual threshold, and the dynamic range of 10,000:1 used for the evaluation of the Barten model in Section 2 cor-

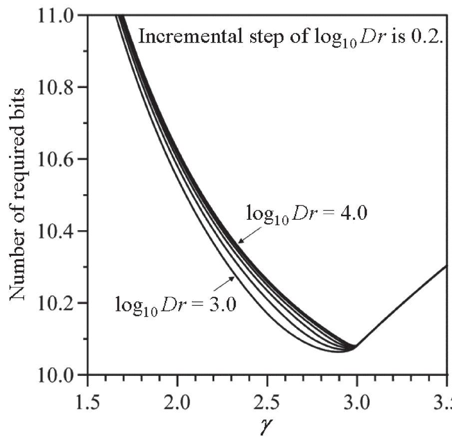


Fig. 6 Required bit depth from color difference model.

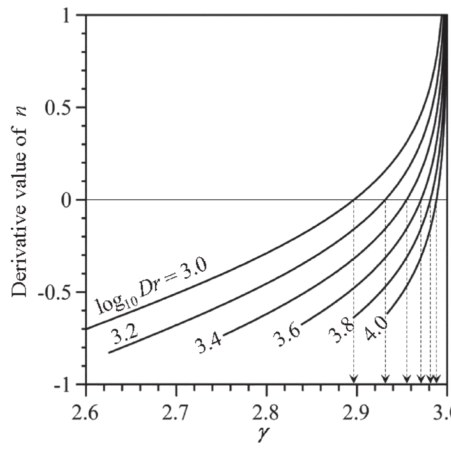


Fig. 7 Gamma value that gives minimum bit depth from color difference model.

responds to D = 4.0. This result shows that the bit depth greater than 10 bits is enough to avoid pseud-contours in color images.

The value of γ that gives the minimum quantization bit depth is different for various dynamic ranges, and this value is obtained by differentiating Eq. (48) with γ and searching γ that gives the minimum value of n. There is no essential meaning in the ordinate value in Fig. 7 though it shows the change in the differential coefficient of n with γ. It can be seen from the figure that the value of γ at which the differential coefficient becomes 0 (i.e., n is minimized) lies in the range of 2.9 to 3.0. For example, if we assume the optimum gamma value is γ = 2.9, the required bit depth derived from the Barten model for grayscale images in Fig. 5 (n is about 11 bits) is about one bit greater than that from the CIE 1976 $L^*a^*b^*$ color difference model (n is about 10 bits).

5. CONCLUSION

In this paper, we examined theoretically the relationship between the required quantization bit depth and the gamma value for DCDM assuming both the Barten model as a perceptual model for grayscale images and the CIE 1976 $L^*a^*b^*$ color space as a perceptual model for color images. The criterion used for avoiding pseud-contour is that the luminance change or color difference induced by one gamma-quantization step change becomes smaller than one JND. As a result, the required bit depth obtained from the Barten model decreases gradually as the gamma value increases, and the required bit depth obtained from the CIE 1976 $L^*a^*b^*$ color difference model is minimized when the gamma value is 2.9 to 3.0. Moreover, the required quantization bit depth derived from the Barten model based on the grayscale images is about one bit greater than that obtained from the CIE 1976 $L^*a^*b^*$ color difference model based on color images at γ = 2.9.

The Barten model is widely accepted as a contrast sensitivity function for grayscale images, while the CIE 1976 $L^*a^*b^*$ model does not supply a complete uniform color difference. Actually the improvement has been done for the color difference models such as the CIE 1994 color difference model and CIE2000 color difference formula [8], etc. The reason why we used the CIE 1976 $L^*a^*b^*$ model in this paper is that it is easy to analyze the color difference compared to other uniform color space models. Although a result obtained here shows that the required quantization bit depth for color images is not severe against that for the grayscale images, it can be thought that this result is significant to some degree because actual digital cinema employs color images. Moreover, it is verified that the requirement for DCDM bit depth of 12 bits, which is determined based on the subjective evaluation experiment, is sufficient based on the theoretical considerations using contrast and color different sensitivities.

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